In memory of our colleague and friend
Ary Belyshev who was the initiator of our common works

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About the illusions and the reality in the methods of probability analysis of vector hydrometeorological processes

It should not treat indifferently to the choice
of one or other mathematical method because
different methods are not entirely equivalent
in terms of their relation to the reality.
G. Kazanova. Vector algebra.

In 1982 we have published in “Труды ГОИН” (“Transactions of the State Oceanographic Institute”, or SOI) the article: “A. Belyshev, Yu. Klevantsov, V. Rozhkov. About the illusions and reality in the methods of sea current analysis” [4].

Nearly 30 years have passed, but we found no one article in foreign editions where the authors used the “vector-algebraic method” for vector process analysis (it was elaborated during 1974-1978 in Leningrad Branch of SOI and then published in 1983 (in Russian) in the monograph “Probabilistic analysis of sea currents” [5]). So we decided to repeat the publication of the basic ideas of that old article in English in the hope that somebody will read it with some interest and even will try to apply this method for the statistical analysis of his own vector data. It is difficult now for us to make the translation of the book [5] and to place it in our web-site; so we tried to adduce some interpretation of invariant characteristics of correlation and spectral functions of current or wind velocity (or gradients of scalar processes) very briefly; may be in future we’ll be able to translate and publish in more detail the peculiarities of this interpretation.

The data of long measurements of sea current or wind velocity are one of the basic information source about the regularities of structure and dynamics of these processes. The data may be received in some fixed point or in several points (in particular on the polygon or in the model net).

For the obtaining of sea current or wind statistical regularities these data are considered as vector probabilistic process $\mathbf{V}(t)$; its characteristics are the moment functions of the first and the second order .

(Here and further we notice the vector (in contradistinction to the scalar) value, function or process by heavy-faced type).

The space-time characteristics of this process were computed on the base of auto- and cross-correlation and spectral analysis of velocity time series which were measured on the different horizons of one or several stations.

As a first approximation process **V**(t) is called two-dimensional (though the modern devices allow to receive three-dimensional data; the analysis of current velocity vertical component shows that it can have not so small values compared some time with horizontal component values, and we elaborated now the method of vector-algebraic analysis for such data).

In the applied works often uses the examination of two-dimensional vector as

- well regulated number couple (abstract vector coordinates in some basis) [21, 23];
- complex number, real and imaginary parts of which coincide with Cartesian projections of vector [1, 23];
- the value, which has a modulus, a direction and can be added with the same values by the "rule of parallelogram".

For sea currents in oceanography are four approaches to the analysis of current velocity; call them

- "component-wise";
- "complex-valued" and "method of rotary components";
- "vector-algebraic method";

everyone of these three position may be correlated with three above-mentioned definitions of vector.

Two of them ("component-wise" and "vector-algebraic method") are use now for wind velocity probabilistic analysis although (for example [8, 17, 21].

In "component-wise" analysis [22]:

- orthogonal components of vector **V** are considered as system of two random values Vx and Vy;
- the expectation of stationary vector process – as the number couple (m(Vx), m(Vy));
- correlation function K(τ) and spectral density S(ω) by the fixed significance of argument τ or ω - as the matrix

$$\begin{pmatrix} \mathrm{K}_{\mathrm{V}\,x}(\tau) \\ \mathrm{K}_{\mathrm{V}\,y}(\tau) \end{pmatrix} \text{ or } \begin{pmatrix} \mathrm{S}_{\mathrm{V}\,x}(\omega) \\ \mathrm{S}_{\mathrm{V}\,y}(\omega) \end{pmatrix}$$

Such approach quite justified for abstract vectors in n-dimensioned space; it assumed joint analysis of all matrix components. It is shown in [2] that in this case (when the coordinates of abstract vector coincide with Cartesian projections) the elements of matrix give us only the initial, “raw” material, and for the receiving of pithy conclusions about the properties of analyzing process it is necessary to introduce the operations of computation of some invariant values. Farther still such probabilistic characteristics as coherent function and spectral phase between the projections of current velocity are deprived of physical sense; the moment functions of projections depend both from the properties of process and from the orientation of coordinate axes, that is in the general case they are non-invariant values.

The methods of analysis of current and wind velocity as complex functions are correct only in the limits which are determined by the specification of the multiplication and Fourier transformation operations; if you not introduce additional assumptions the results will be erroneous.

The method of “rotary components” [27, 30, 31] introduce additional restriction: it attribute the analyzing process to the class of polarized fluctuations. But the assumption that current or wind velocity fluctuations are polarized fluctuations (algorithm of statistic characteristics of polarization is cited in [25]) is not well founded physically, so far as these fluctuations inherent only to transverse waves [12, 20].

The “vector-algebraic method” [2, 3, 5, 8] follows from most general properties of vectors in ordinary physical space; it uses as probabilistic characteristics such values:

- vector of expectation;
- correlation tensors;
- spectral density tensors.

As the elements of all these tensors are not invariant values in the general case, we use the combinations of elements which named “the invariants of tensor’.

The success of analysis of each physical phenomenon or process essentially depends as far as how mathematical method reflects the properties of real object.

The current is “progressive motion of water masses” (so as the wind flow is “progressive motion of air masses”); therefore the approach to the current analysis trough the vector of transference is the most physically grounded, and analysis method is most argued, if it is based on the vector algebra lows

So far as the number of publications increases where “component-wise” approach and “rotary component method” is used for analysis of current velocity in Russian and foreign editions, the authors of present article put up the task before them – to pay attention of readers to the limitation of conclusions which can be received by “component-wise” analysis of elements of correlation function and spectral density function matrix of current or wind velocity projections. Also we are writing about the illegality of this analysis result interpretation and about the erroneous interpretation of sea current velocity probabilistic characteristics which were received by “rotary component method”.

In order to decide this task consider the principal probabilistic characteristics of sea current (or wind) velocity which are received by different methods and compare these characteristics take for the base “vector-algebraic method”.

1. <u>The expectation</u> of current velocity in assumption of process stationary is defined as vector

$$\mathbf{m_V} = M\{\mathbf{V}(t)\} = M\{ v_1(t)\, \mathbf{e}_1 + v_2(t)\, \mathbf{e}_2 \} , \tag{1}$$

where M – expectation operator; $v_1(t)$ and $v_2(t)$ – current velocity projections by the direction of basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$.

Value $\mathbf{m_V}$ characterizes mean value of velocity (its modulus and direction) of water mass transference in fixed point of ocean.

In “component-wise” analysis the expectation is considered as well regulated number couple m_{v1} , m_{v2} , which in the general case are non-invariant values and have no clear physical sense (m_{v1} is mean value of transference along axis $\mathbf{e}_1$, and m_{v2} - the same along axis $\mathbf{e}_2$.

When vector process $\mathbf{V}(t)$ is replaced by complex valued

$$V(t) = v_1(t) + iv_2(t) \tag{2}$$

the expectation of V(t) is complex value

$$m_V = m_{v1} + im_{v2} , \tag{3}$$

which is interpreted as vector on complex plane.

We shall consider for simplification of further calculations that process $\mathbf{V}(t)$ is centered random process, that is assume $\mathbf{V}^o(t) \equiv \mathbf{V}(t)$.

2. <u>Correlation function</u> of sea current velocity is defined [2] as the expectation of tensor product:

$$K_{\mathbf{V}}(\tau) = M\{\mathbf{V}(t) \otimes \mathbf{V}(t+\tau)\}, \tag{4}$$

where $\otimes$ is tensor product operator.

Function $K_{\mathbf{V}}(\tau)$ characterizes intensity of current velocity directional alterations, their orientation in set coordinate system and correlation of these alterations on every interval τ; by fixed τ value $K_{\mathbf{V}}(\tau)$ is correlation tensor of the second rank with matrix

$$K_{\mathbf{V}}(\tau) = \begin{pmatrix} K_{v1v1}(\tau), & K_{v1v2}(\tau) \\ K_{v2v1}(\tau), & K_{v2v2}(\tau) \end{pmatrix}. \quad (5)$$

Note that each matrix component of tensor $K_{\mathbf{V}}(\tau)$ in the general case is non-invariant value.

The system of current velocity correlation functions is considered in "component-wise" approach; by each fixed value τ $K_{\mathbf{V}}(\tau)$ is correlation matrix in the form of (5) and each element of matrix has independent role: $K_{v1v1}(\tau)$ and $K_{v2v2}(\tau)$ – correlation functions of projections $v_1(t)$ and $v_2(t)$;

$K_{v1v2}(\tau)$ and $K_{v2v1}(\tau) = -K_{v1v2}(\tau)$ - cross-correlation function of these projections. In "vector-algebraic method" we use the invariants of tensor (5). The expectation of scalar product of vectors $\mathbf{V}(t)$ and $\mathbf{V}(t+\tau)$ can be put up in conformity to the linear invariant $I_1(\tau)$ of tensor $K_{\mathbf{V}}(\tau)$:

$$I_1(\tau) = K_{v1v1}(\tau) + K_{v2v2}(\tau) \Rightarrow M\{\mathbf{V}(t)\bullet\mathbf{V}(t+\tau)\}, \quad (6)$$

where • - the operator of scalar product of vectors; => - conformity sign.

Hence linear invariant characterizes general intensity of alterations which not depends on what is changed – modulus or direction of current (or wind) velocity.

Taking into account the definition of vectors $\mathbf{V}(t)$ and $\mathbf{V}(t+\tau)$ collinearity in the form of

$$\mathbf{V}(t)\bullet\mathbf{V}(t+\tau) = |\mathbf{V}(t)|\,|\mathbf{V}(t+\tau)|\,, \quad (7)$$

we can interpret linear invariant

$$I_1(\tau) = M\{\,|\mathbf{V}(t)|\,|\mathbf{V}(t+\tau)|\cos(\mathbf{V}(t)\wedge\mathbf{V}(t+\tau)\} \quad (8)$$

as the measure of collinear correlation between current vectors on interval τ; in the strict sense we must define the measure of collinear correlation on interval τ as the ratio $I_1(\tau)/K_{|V|}(\tau)$ of linear invariant to correlation function of current velocity modulus.

Note that

$$I_1(0) = K_{v1}(0) + K_{v2}(0) \Rightarrow M|\mathbf{V}^2(t)| \equiv M\{|\mathbf{V}(t)|^2\}\,, \quad (9)$$

where $\mathbf{V}(t)$ mean $\mathbf{V}^o(t)$;

but the square of modulus $\mathbf{V}^o(t)$ in (9) is not identical to the variance of modulus of initial process $\mathbf{V}(t)$, as

$$V^0(t) = \begin{pmatrix} v_1(t) - m_{v_1(t)} \\ v_2(t) - m_{v_2(t)} \end{pmatrix}$$

$$|\mathbf{V}(t)|^o = |\mathbf{V}(t)| - M\{|\mathbf{V}(t)|\} .$$

Main or own values $\lambda_1(\tau)$ and $\lambda_2(\tau)$ of tensor (5)

$$\lambda_{1,2}(\tau) = 0.5[K_{v1}(\tau) + K_{v2}(\tau) \pm \{[K_{v1}(\tau) - K_{v2}(\tau)]^2 + [K_{v1v2}(\tau) + K_{v2v1}(\tau)]^2\}^{0.5}] \qquad (10)$$

characterize (by each τ) extreme values of current velocity projection correlation functions along the orthogonal directions. These values may be although interpreted as longer (major) and shorter (minor) semi-axes of tensor curve of the second order which put up in conformity throw the invariant

$$I_2(\tau) = \lambda_1(\tau) \bullet \lambda_2(\tau) \qquad (11)$$

of symmetric part of tensor (5).

The invariant $\Omega(\tau)$ of anti-symmetric part of tensor (5) may be taken for indicator of rotary motions which are peculiar to process $\mathbf{V}(t)$; it is defined in the form of

$$\Omega(\tau) = K_{v1v2}(\tau) - K_{v2v1}(\tau) \Rightarrow M\{\mathbf{V}(t) \times \mathbf{V}(t+\tau)\} , \qquad (12)$$

where x – the operator of "oblique" product of vectors. The invariant $\Omega(\tau)$ (which is interpreted as the expectation of "oblique" product) characterizes the modulus of current velocity orthogonal alternations on interval τ. The sign of $\Omega(\tau)$ defines general orientation of rotary motions (prevailing direction of turning from one vector $\mathbf{V}(t)$ to another $\mathbf{V}(t+\tau)$.

In linear algebra the operator of "oblique" product is interpreted as operator of turning on 90° [13]. In particular, it follows from here that value

$$\frac{\Omega(\tau)}{K_{|V|}(\tau)} = \frac{M\{|V(t)||V(t+\tau)|\sin\{V(t) \wedge V(t+\tau)\}}{M\{|V(t)||V(t+\tau)|\}}$$

characterizes the measure of orthogonal correlation between the current velocity values on interval τ.

If $\Omega(\tau) > 0$, then vector rotation occurs mostly clockwise; if $\Omega(\tau) < 0$ – counterclockwise. If $\Omega(\tau) = 0$, this testifies about the absent of regularity (prevailing mono-directness) of rotary alternations, as it is possible the situation when the intensity of velocity rotary alternations clockwise and counterclockwise are equal. The equality $I_1(\tau) = \lambda_1(\tau)$ is necessary and sufficient condition of velocity rotary alternation absence. Note that tensor product operation is not commutative, therefore correlation function $K_{\mathbf{V}}(\tau)$ is not even-numbered; in particular this follows from the comparison of expression (11) for $\Omega(\tau)$ and $\Omega(-\tau)$, as $\Omega(-\tau) = -\Omega(\tau)$. This property of function $K_{\mathbf{V}}(\tau)$ will be used further by the discussion of properties of spectral density.

Orientation of major axis of tensor curve is defined for fixed τ by n0n-invariant value

$$\alpha(\tau) = 0.5\,\mathrm{arctg}\ \frac{K_{v1v2}(\tau) + K_{v2v1}(\tau)}{K_{v1v1}(\tau) - K_{v2v2}(\tau)} \tag{13}$$

Value (13) characterize the orientation of orts of main axes $\mathbf{e}_1$ and $\mathbf{e}_2$ of tensor curve relatively the basis $\mathbf{ee}_1$ and $\mathbf{ee}_2$ of initial coordinate system which is chosen for computation of the projections $v_1(t)$ and $v_2(t)$ of vector process $\mathbf{V}(t)$. the value $\alpha(\tau)$ facilitates the interpretation of tensor $K_{\mathbf{V}}(\tau)$ for its notion in the form

$K_{\mathbf{V}}(\tau) = \lambda_1(\tau)(\mathbf{e}_1 \text{ x } \mathbf{e}_1) + \lambda_2(\tau)(\mathbf{e}_2 \text{ x } \mathbf{e}_2) - 0.5\,\Omega(\tau)[(\mathbf{e}_1 \text{ x } \mathbf{e}_2) - (\mathbf{e}_2 \text{ x } \mathbf{e}_1)]$. (14)

The expression (14) shows the sufficiency of the tensor $K_{\mathbf{V}}(\tau)$ analysis through three scalar functions $\lambda_1(\tau)$, $\lambda_2(\tau)$, $\Omega(\tau)$; but their application not abolishes the necessity of using the invariants $I_1(\tau)$, $I_2(\tau)$ and other for description of process $\mathbf{V}(t)$ properties.

Note the limits of suitableness of correlation matrix element component-wise analysis, based on the definitions (8) – (14).

The results of vector process analysis in natural (vectorial) form must be formulated although in this form, in particular relatively of the product operations. The analysis of correlation tensor invariant characteristics completely satisfied this requirement. The attraction of additional operations to the analysis (in particular vector transformation in coordinate form) results to the computation of correlation matrix; its elements are the intermediate result; the algorithm of further transformation of matrix elements in such values (the correlation tensor invariants) which assists to reveal the analyzing vector process properties is required.

The principal information sense follows from the analysis of $I_1(\tau)$ and $\Omega(\tau)$; other characteristics serve for extending of separate conclusions. Hence only in particular case of diagonal correlation matrix its elements coincide with invariants $\lambda_1(\tau)$ and $\lambda_2(\tau)$ and component-wise results coincide with results of "vector-algebraic method"; in this case $\mathbf{e}_i = \mathbf{ee}_i$.

Correlation function of complex stationary process has a form

$K_V(\tau) = M\{V(t)\, V^*(t+\tau)\}, = K_{v1v1}(\tau) + K_{v2v2}(\tau) + i[K_{v2v1}(\tau) - K_{v1v2}(\tau)]$, (15)

where * is the sign of complex conjugating.

If take into account (6) and (12), expression (15) would be expressible in form

$$K_{\mathbf{V}}(\tau) \Rightarrow I_1(\tau) + i\Omega(\tau), \tag{16}$$

that is invariant characteristics of tensor $K_{\mathbf{V}}(\tau)$ put in conformity to the real and imaginary parts of correlation function (15); this gives a reason for sense analysis of dependence type between the correlated values. But in (15) there is no information about $\lambda_1(\tau)$ and $\lambda_2(\tau)$, hence about $I_2(\tau)$, what makes an analysis more poor. Moreover the value $K_{\mathbf{V}}(\tau)$ in (4), (5) has "more complicated structure than the vector" [19] by the tensor product; value $K_V(\tau)$ in (15) by this product operation is (by fixed τ) also complex number as the initial values V(t). However if the interpretation of V(t) through to a certain degree was permissible, then analogous interpretation of $K_V(\tau)$ will be erroneous; this is the direct consequence of non-isomorphism of operations of vector **V**(t) and vector **V**(t+τ) tensor product and complex number product.

The error possibility consists in particular in the nonconformity of "complex vector projections" $K_V(\tau)$ and invariant characteristics $I_1(\tau)$ and $\Omega(\tau)$.

Cross-correlation function of two current velocity time series **V**(t) and **U**(t) is defined in [2] as the expectation of vector **V**(t) and **U**(t) tensor product $\mathbf{K_{VU}}(\tau) = M\{\mathbf{V}(t) \otimes \mathbf{U}(t+\tau)\}$.

The function $\mathbf{K_{VU}}(\tau)$ defines mutual community of intensity of current velocity directed changes in set coordinate system on interval τ; by fixed τ value $\mathbf{K_{VU}}(\tau)$ is correlation tensor of the second rank with matrix

$$K_{\mathbf{VU}}(\tau) = \begin{pmatrix} K_{v1u1}(\tau), & K_{v1u2}(\tau) \\ K_{v2u1}(\tau), & K_{v2u2}(\tau) \end{pmatrix}.$$

The invariant characteristics of cross-correlation tensor $K_{\mathbf{VU}}(\tau)$ are analogous considered above characteristics of auto-correlation tensor $K_{\mathbf{V}}(\tau)$. All stated above about the result comparison of component-wise correlation analysis and correlation function of complex random process is fair in the same degree for auto- and cross-correlation analysis; in this article it is not considered.

3. <u>Spectral density</u> of current velocity is the Fourier transformation of correlation function $K_{\mathbf{V}}(\tau)$:

$$S_{\mathbf{V}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau} K_V(\tau)\, d\tau. \tag{17}$$

It characterizes the distribution of intensity of current velocity directed fluctuations by the frequencies ω_k and their orientation in set coordinate system. By each fixed ω it although is a tensor of the second rank with matrix

$$S_V(\omega) = \begin{pmatrix} S_{v_1v_1}(\omega) & S_{v_1v_2}(\omega) \\ S_{v_2v_1}(\omega) & S_{v_2v_2}(\omega) \end{pmatrix} = \begin{pmatrix} S_{v_1v_1}(\omega) & C_{v_1v_2}(\omega) + iQ_{v_1v_2}(\omega) \\ C_{v_1v_2}(\omega) - iQ_{v_1v_2}(\omega) & S_{v_2v_2}(\omega) \end{pmatrix} \tag{18}$$

The elements of matrix (18) are the Fourier transformations of matrix (5) elements and hence in common case are non-invariant values.

For the convenience of further consideration will offer the tensor (18) in view

$$S_V(\omega) = \begin{pmatrix} \lambda_1(\omega) & 0 \\ 0 & \lambda_2(\omega) \end{pmatrix} + \begin{pmatrix} 0 & 0.5\Omega(\omega) \\ -0.5\Omega(\omega) & 0 \end{pmatrix} \qquad (19)$$

where $\lambda_1(\omega)$, $\lambda_2(\omega)$ and $\Omega(\omega)$ – invariant characteristics, analogous to the considered above $\lambda_1(\omega)$, $\lambda_2(\omega)$ and $\Omega(\omega)$ in (10) and (12).

It follows from (19) taking into account the marked above properties of function $K_{\mathbf{V}}(\tau)$ (15, 16)) that $S_{\mathbf{V}}(\omega)$ by each fixed ω is complex value. To its real part it is possible to put into the conformity the linear invariant $I_1(\omega)$ and to the imaginary part – invariant $\Omega(\omega)$. To carry out the analogy which is used by the interpretation of values $I_1(\tau)$ and $\Omega(\tau)$ it is possible to decide that $I_1(\omega$ and $\Omega(\omega)$ characterize the intensity of collinear and orthogonal process variability in frequency field; symbol i , as follows from (17), separate cosine- and sine-Fourier transformations.

The interpretation of the invariants of tensor (18) on the whole is analogous to the interpretation of tensor (15). Draw your attention to the fact that $I_1(\omega) \geq 0$ and “rotation indicator” $\Omega(\omega)$ always is imaginary number

$$\Omega(\omega) = 2iQ(\omega) \qquad (20)$$

The absolute value of this invariant characterizes the intensity of current velocity rotation in analogous range of frequencies and its sign mean the rotation direction (“+” – to the right, the clockwise; :-: - to the left, counterclockwise direction).

For sea currents or wind always takes place severe inequality $I_1(\omega) > \Omega(\omega)$ except the trivial case $I_1(\omega) = \Omega(\omega)$ -= 0.

So far as spectral density $S_{\mathbf{V}}(\omega)$ is defined in form (17) as Fourier-image of correlation function $K_{\mathbf{V}}(\tau)$, all marked above defects of component-wise analysis of matrix $K_{\mathbf{V}}(\tau)$ to the equal degree concern to the component-wise analysis of matrix of spectral density (18) of $\mathbf{V}(t)$ projections.

Spectral density of complex stationary process has a form

$$S_V(\omega) = S_{v1v1}(\omega) + S_{v2v2}(\omega) + S_{v1v2}(\omega) - S_{v2v1}(\omega) \qquad (21)$$

Formula (21) expresses well-known fact that spectral density both for the real and for the complex scalar processes always is real. This property of Fourier transformation of scalar processes although is limiting factor when are discussed the boundaries of suitableness of complex number notion when are studied the properties of vector processes. Recall that in (2) symbol i served as indicator of the difference of initial process orthogonal projections. Presence of i in (15) already led to the difficulties in comparison of correlation functions of vector and complex processes. “Disappearance” of i in (21) makes non-comparing the spectral densities of processes **V**(t) and $V(t) = v_1(t) + iv_2(t)$, that is refutes the interpretation possibility of complex process spectrum in terms of tensor (18) invariants.

The apparent exit from arising "deadlock" was found by the authors of "method of rotary components" [30], who began to examine he spectral components W(ω) of analyzing process as superposition of polarized components w_1 and w_2 with different rotation direction. These components were presented in form of complex conjugating numbers

$$w_1 = |W|\exp(iQ); \quad w_2 = |W|\exp(-iQ),$$

where Q is quadrature (squaring) part of the cross-spectrum of velocity projections.

As the basic probability characteristics of this process were chosen:

- the spectra of rotary motions

clockwise

$$S^{-}(\omega) = 0.125[S_{v1v1}(\omega) + S_{v2v2}(\omega) - 2Q_{v1v2}(\omega)] \tag{22}$$

and counterclockwise

$$S^{+}(\omega) = 0.125[S_{v1v1}(\omega) + S_{v2v2}(\omega) + 2Q_{v1v2}(\omega)] \; ; \tag{23}$$

total spectrum

$$S_t(\omega) = 0.25[S_{v1v1}(\omega) + S_{v2v2}(\omega)] \; ; \tag{24}$$

difference of spectra

$$S\_(\omega) = 0.5Q_{v1v2}(\omega)] \; ; \tag{25}$$

rotation coefficient

$$C_R(\omega) = \frac{-2Q_{V_1V_2}(\omega)}{S_{V_1V_1}(\omega) + S_{V_2V_2}(\omega)} \tag{26}$$

mean ellipse orientation

$$\tan 2\psi(\omega) = \frac{2Co_{V_1V_2}(\omega)}{S_{V_1V_1}(\omega) - S_{V_2V_2}(\omega)} \tag{27}$$

Stability of ellipse orientation

$$|E(\omega)|^2 = \frac{\left[S_{V_1V_1}(\omega) + S_{V_2V_2}(\omega)\right]^2 - 4\left[S_{V_1V_1}(\omega)\,S_{V_2V_2}(\omega) - Co^2_{V_1V_2}(\omega)\right]}{\left[S_{V_1V_1}(\omega) + S_{V_2V_2}(\omega)\right]^2 - 4Q_{V_1V_2}(\omega)} \tag{28}$$

First of all note that these characteristics were introduced for description of some properties of electromagnetism and optical phenomena, since only transverse waves have initial natural property – wave polarization.

The reference of sea currents and wind (fluctuations or waves) to the class of transverse waves is not well founded physically, so far as the velocity fluctuations on the field of sea currents or wind occur in the main in horizontal plane that is in the plane of their spreading [16]. That is the velocity changes with more ground must be considered as longitudinal waves. Hence it follows that the question about the using justice of “method of rotary components” for sea currents or wind analysis is highly discussing. For more severe definition of use sphere limits of this method we shall compare the characteristics (22)-(28) with some spectral tensor (17) invariants.

At first note that the same quantitative values of $S^+(\omega)$ and $S^-(\omega)$ can be received not only by superposition of two circular polarized fluctuations but as well as by superposition of:

- two linear orthogonal polarized fluctuations both with equal and with different amplitudes;
- circular or elliptic and linearly polarizing fluctuations;
- two elliptic polarized fluctuations, as well as even one elliptic (not without fail polarized) fluctuation [12, 20].

It in particular follows that tide currents of definite cyclic recurrence whose velocity changes by elliptic low and at one-directional rotation can have different from zero both $S^+(\omega)$ and $S^-(\omega)$ simultaneously. It is impossible to explain this fact physically.

The name “total spectrum “ assumes that terminologically that this spectrum characterizes the totality of every kind (by modulus and by direction) changes of vector process, that is it is completely equivalent to tensor (17) of spectral density. But definition (24) coincides with the definition of invariant $I_1(\omega)$ with the accuracy to the constant multiplier.

The invariant $\Omega(\omega)$ corresponds to the spectra difference $S_-(\omega)$; but interpretation of $S_-(\omega)$ is essentially became poor by the definition of this value through the “mean area of elliptic surfaces”, moreover the reader must guess – which surfaces.

Spectral invariants of rotation (22) and (23) are constructed by the means the summing up of the values belonging to the real and imaginary parts of spectral tensor, as it is seen from (19); that is here has a place the same “paradox” as in formula (21). The loss of symbol i leading to the marked “paradox” in (22) and (23) has a place although in (25).

The interpretation of (26) is correct only partly. Really by cleanly rotary motion $C_R(\omega) = 1$, but yet equality $C_R(\omega) = 0$ quite not means one-directional motion, but only the absence of regular rotation in the variability of vector process. For example for the case of isotropic turbulence “rotation indicator” $\Omega(\omega) = 0$.

The expression (27) is identical to the major axis orientation $\alpha(\omega)$ of characteristic ellipse of tensor (19) symmetrical part (spectral density tensor of velocity) [2]

$$\alpha(\omega) = 0.5 \text{ arctg } \{2Co_{v1v2}(\omega) / [S_{v1v1}(\omega) - S_{v2v2}(\omega)]\}. \quad (29)$$

However the interpretation of equations (27) and (29) is different, hence they concern to the ellipses of different origin. It is impossible to consider as well founded the interpretation of invariant value $|E(\omega)|^2$ as "stability of ellipse orientation" and so much the coherence. By the way of reducing (28) to the main axes we shall receive

$$|E(\omega)|^2 = [\lambda_1(\omega) - \lambda_2(\omega)]^2 / \{[\lambda_1(\omega) + \lambda_2(\omega)]^2 - 4Q^2_{v1v2}(\omega)\} \quad (30)$$

Yet the invariants $\lambda_1(\omega)$ and $\lambda_2(\omega)$ are the main axes of characteristic ellipse of velocity spectrum but the invariant $2iQ_{v1v2}(\omega)$ by no means is connected with ellipse, that is the value (28) has not clear geometrical interpretation.

We shall dwell on the notion of coherence of two vector processes separately. The idea of "total" or "general" coherence is conditioned from the correlation

$$\gamma^2_{\mathbf{VU}}(\omega) = [\mathbf{S}_{\mathbf{VU}}(\omega)]^2 / [S_{\mathbf{V}}(\omega)*S_{\mathbf{U}}(\omega)] , \quad (31)$$

that is (31) although is the tensor of the second rank which we can call "coherence tensor". More simple (but naturally owning lesser than (31) community) definition of coherence can be given through the ratio of the square of cross-spectrum linear invariant to the product of auto-spectra linear invariants of analyzing rows

$$\mu^2_{\mathrm{VU}}(\omega) = \frac{\left[\mathrm{Co}_{\mathrm{V}_1U_1}(\omega) + Co_{\mathrm{V}_2U_2}(\omega)\right]^2 + \left[\left|\mathrm{Q}_{\mathrm{V}_1U_1}(\omega)\right| + \left|\mathrm{Q}_{\mathrm{V}_2U_2}(\omega)\right|\right]^2}{\left[\mathrm{S}_{\mathrm{V}_1V_1}(\omega) + \mathrm{S}_{\mathrm{V}_2V_2}(\omega)\right]*\left[\mathrm{S}_{\mathrm{U}_1U_1}(\omega) + \mathrm{S}_{\mathrm{U}_2U_2}(\omega)\right]} \quad (32)$$

In [2] analogous expression was named "coherence" and in [6] formula (32) was given with a misprint – instead of quadrature part of cross-spectrum linear invariant was placed co-phase part of "indicator of rotation". Note that the analysis of expressions (22)-(25) was executed not taking into account the number coefficients in these formulas.

Thus on the basis of this analysis we can make an inference about the groundlessness of the employment of "method of rotary components" to the analysis of current velocity. G. Kazanova in [15] pays attention to the similar situation; E.E. Slutsky [26] speaks against "the mechanical transfer of the methods growing on the another ground".

Let us consider some problems of result interpretation of spectral analysis of current velocity which was measured on buoy stations in different regions of World Ocean.

The first of them was executed in near Rockoll Bank in North Atlantic and analyzed by “component-wise method” in [22]; the second was executed in Atlantic ocean and analyzed in [10, 11] by the “method of rotary components”; another were executed in central part of the Baltic Sea during the international survey 1964 [28] and analyzed by “vector-algebraic method” by authors of this article.

The results of spectral analysis on the first station are offered in invariant form on fig.1. It is seen that current velocity variability is conditioned by four cyclic recurrences: semi-diurnal and diurnal tide and two in zone of synoptic fluctuations (division of them was possible only thanks to different direction of their rotation). Resolve ability for spectral analysis for realization of such duration is insufficient for the division of these frequencies on the graphs $I_1(\omega)$ and $\lambda_{1,2}(\omega)$ in synoptic band of frequencies. The change intensity is maximal on the frequency 0.025 radian/hour (cyclic recurrence equal roughly 252 hour or 10.5 days) by the velocity rotation clockwise mainly; intensity of changes on ω = 0.075 rad/h (cyclic recurrence equal roughly 84 hours or 3.5 days) obviously lesser by the rotation counterclockwise mainly.

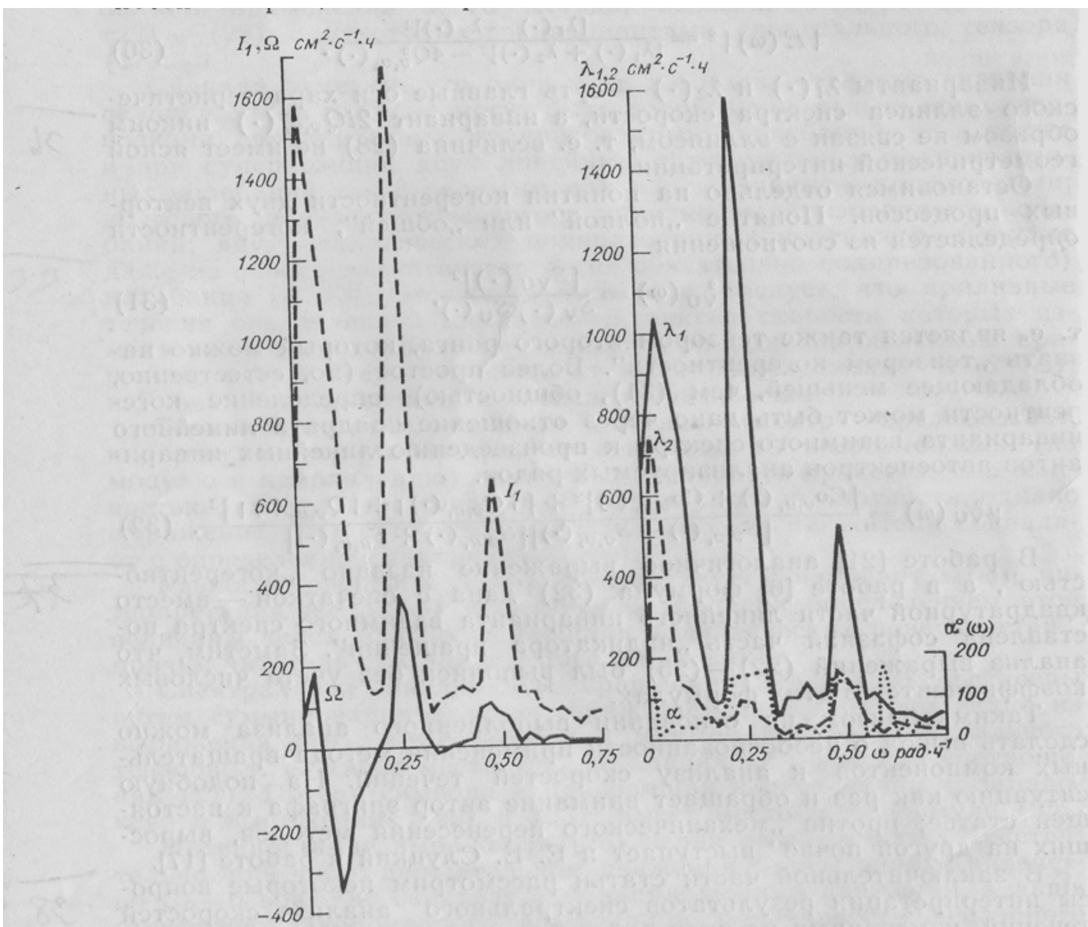


Fig. 1. Spectral tensor invariants for ABS on Rockol Bank, 1958, z = 400 m. T = 708 hours, $\Delta\omega$ = 0.0253 rad/hour

The invariants $\lambda_1(\omega)$ and $\lambda_2(\omega)$ complete the information about the current velocity fluctuation structure. For example the proximity of these characteristics on ω = 0.025 rad/hours and comparable small value of invariant $\Omega(\omega)$ testify about essential isotropy of velocity changes and considerable non-orderness of their rotary changes in synoptic range of frequencies. Last regularity completes the degree of order and non-regularity characteristics of process changes by spectrum width in examining frequency field. The comparison of these invariants on frequency 0.075 rad/h indicates on the anisotropic character of the changes along the axis 40-220° and on the more well regulated rotation character, but counter clockwise.

Small values of invariant $\Omega(\omega)$ (as compared with the values in zones of tide fluctuations with frequencies of peaks 0.25 and 0.50 rad/hours) testify about rather large stretchness of tide fluctuation ellipses. The comparison of values $\Omega(\omega)$, $\lambda_1(\omega)$ and $\lambda_2(\omega)$ allows to affirm that the velocity rotation occurs clockwise; the diurnal cyclic recurrence fluctuations more well regulated than semi-diurnal; major axis of ellipse for diurnal component has the orientation along 160-340° and for semi-diurnal – along 130-310°.

This example permits to be convinced of the "component-wise" analysis restriction (which gave the opportunity to reveal the cyclic recurrence of process and to show on the width of energy-carrier, or power-carrier zones) and of the advantage of the current velocity "vector-algebraic method" analysis (which gave more pithy information about the process properties).

On fig. 2 are offered the results of spectral analysis of measurements on this (the first) station by "rotary component method".

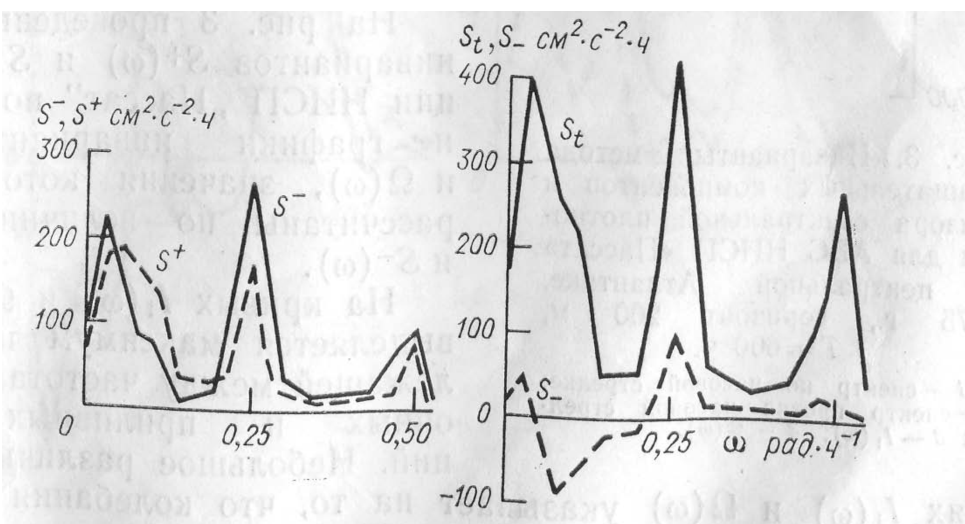


Fig. 2. The invariants of "rotary component method" for ABS on Rockol Bank, 1958, z = 400 m. Parameters are the same as on fig. 1.

The graphs of “total spectrum” $S_t(\omega)$ and “difference of spectra” $S\_(\omega)$, as one would expect, completely repeat the form of the graphs $I_1(\omega)$ and $\Omega(\omega)$ by the decrease absolute values a four times.

The values of “clockwise spectrum” have the maxima on frequencies 0.025; 0.25 and 0.50 rad/h; “counterclockwise spectrum” – on frequencies 0.05; 0.25 and 0.50 rad/h, that is the maxima of rotary components in synoptic fluctuation zone coincide with the spectral density of current velocity on fig. 1. Hence the conclusion in [5] about good “choosing capability” of “rotary component method” for current velocity fluctuation identification on near frequencies to a full degree concern to “vector-algebraic method” although.

The proximity of values of spectral densities $S^-(\omega)$ and $S^+(\omega)$ in all peak (power-carrier) zones from point of view of “rotary component method” is the indicator of reversal character of current velocity changes. This conclusion contradicts to the results of spectral analysis discussing above (by fig. 1).

On fig. 3 are offered the graphs of invariants $S^-(\omega)$ and $S^+(\omega)$ (RS “Passat” station by [10]) and the graphs of invariants $I_1(\omega)$ and $\Omega(\omega)$, whose values were computed by the spectra $S^+(\omega)$ and $S^-(\omega)$.

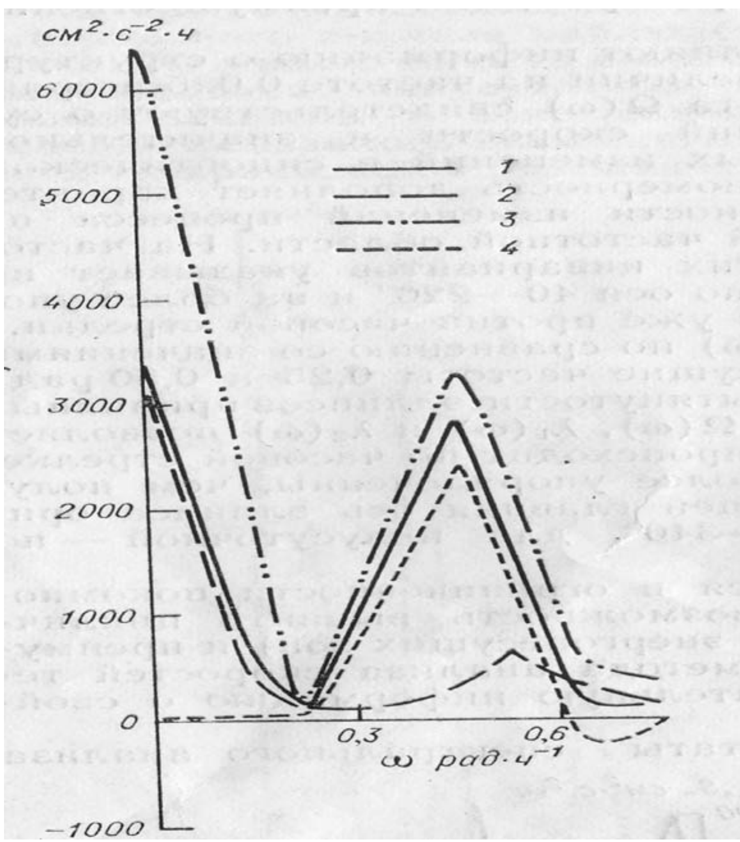


Fig. 3. The invariants of “rotary component method” and of spectral density tensor for ABS Weather Service Ship “Passat” in central Atlantic, 1975, Z = 200 m, T = 600 hours

On the curves $I_1(\omega)$ and $\Omega(\omega)$ is clearly distinguished the maximum on the frequency between the frequencies of inertial and tide fluctuations. Small difference in values $I_1(\omega)$ and $\Omega(\omega)$ indicates that velocity fluctuations are near to rotary type. Positive values of $\Omega(\omega)$ testify about velocity changes clockwise. The maximum on graph of $S^+(\omega)$ coincides by the frequency with the maxima of $I_1(\omega)$ and $\Omega(\omega)$, and maximum of $S^-(\omega)$ is displaced in the field of semidiurnal tide current frequencies. The conclusion about the possibility to divide the tide and inertial fluctuation of current velocity was done from this fact in [10] and [11]. Such conclusion is “debatable”, so far as the spectra $S^+(\omega)$ and $S^-(\omega)$ by the current velocity analysis are deprived of physical sense. Moreover the displacement by the frequency was only at $S^-(\omega)$ on the tide frequency, but spectrum $S^+(\omega)$ didn’t displaces on the inertial fluctuation frequency.

On fig. 4 are offered the spectral analysis results of measurements in the Baltic Sea. It is seen from the figure that the spectral density graph has three clearly expressed maxima on the frequencies 0.025; 0.187; 0.431 rad/h that corresponds to the cyclic recurrences ~10 days; ~ 34 and 14.7 hours.

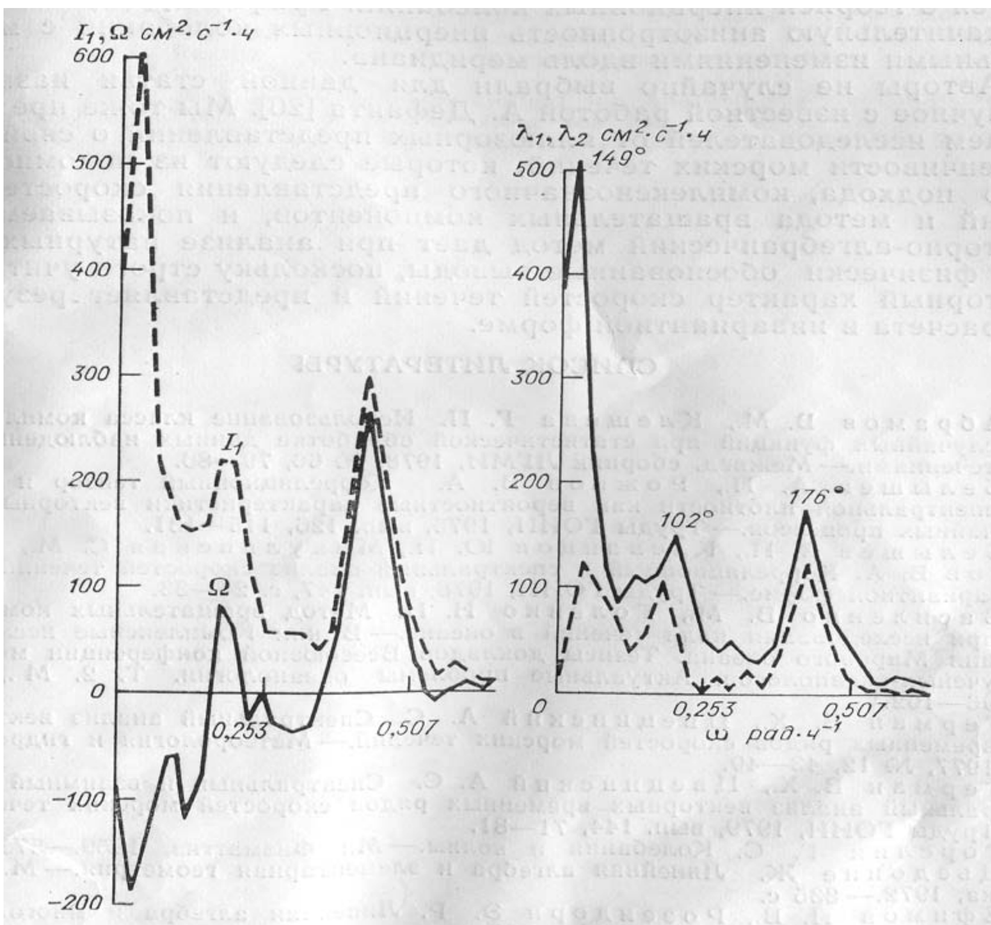


Fig. 4. The invariants of spectral density tensor for ABS in central Baltic.1964, z = 30 m, T = 288 hours, $\Delta\omega$= 0.0253 rad/hour

The last cyclic recurrence coincides with the same of inertial fluctuations on the latitude of measurements. Most intensive 10-diurnal fluctuations have principal counterclockwise rotation by essential anisotropy with maximum changes along the axis 149-329°. 34-hourly changes have least intensity and clockwise rotation by almost full isotropy. Inertial component of velocity changes rotates clockwise by practically constant velocity modulus that confirms with the theory of inertial fluctuations.

Graph $\lambda_{1,2}(\omega)$ shows insignificant anisotropy of inertial fluctuations with maximum changes along the meridian.

For example (and thanks to the fact that we have a place on the site) we take in addition to the old results the nearly full list of statistical analysis of current velocity results (moment characteristics) for one of the longest buoy stations of RS "Rudolf Samoylovich" (AARI); the measurements were made on two horizons (20 and 25 meters) in central part of the Baltic Sea in 1987.

ABS RS-94 had the duration 787 hours. Statistical estimations of probability characteristics of current velocity are adduced in table 1.

From tab. 1 we are seeing:

- mean values on both horizons are very similar one to another (9.4 and 9.2 cm/s, 84° and 73° respectively), only the direction of $\mathbf{m}_{V(t)}$ on Z = 25 m deviates from the direction of $\mathbf{m}_{V(t)}$ on Z = 20 m to the left;
- the values of variance or MSD tensor on 25 m are less than on 20 m;
- the direction of major axis is the same, but the ellipse of variance or MSD on 25 m is a little more stretched (0.66) than on 20 m (0.71); the direction of major axis of variance ellipse is nearly perpendicular to the direction of $\mathbf{m}_{V(t)}$ as on 20 m, so on 25 m, that is the most part of deflection from $\mathbf{m}_{V(t)}$ vectors are directed perpendicularly $\mathbf{m}_{V(t)}$;
- the value of r_s although says that the variable part is more than the quasi-constant part of process.

On fig. 5 and 6 are cited the graphs of correlation and spectral invariants respectively. It is seeing from correlation functions that in that month were prevailed the long synoptic fluctuations with the cyclic recurrences more than 10 days and against their background were small fluctuations with more short cyclic recurrences. It is hard to say about their amplitudes because of the intermitting of these fluctuations.

On spectra graphs we although are seeing one prevailed fluctuation with the peak on frequency of one from synoptic fluctuations and some small peaks on more high frequencies. The little role of directional changes of vectors is seeing on fig. 5 and fig.6 because of small values of "rotation indicator" both $\Omega(\tau)$ and $\Omega(\omega)$.

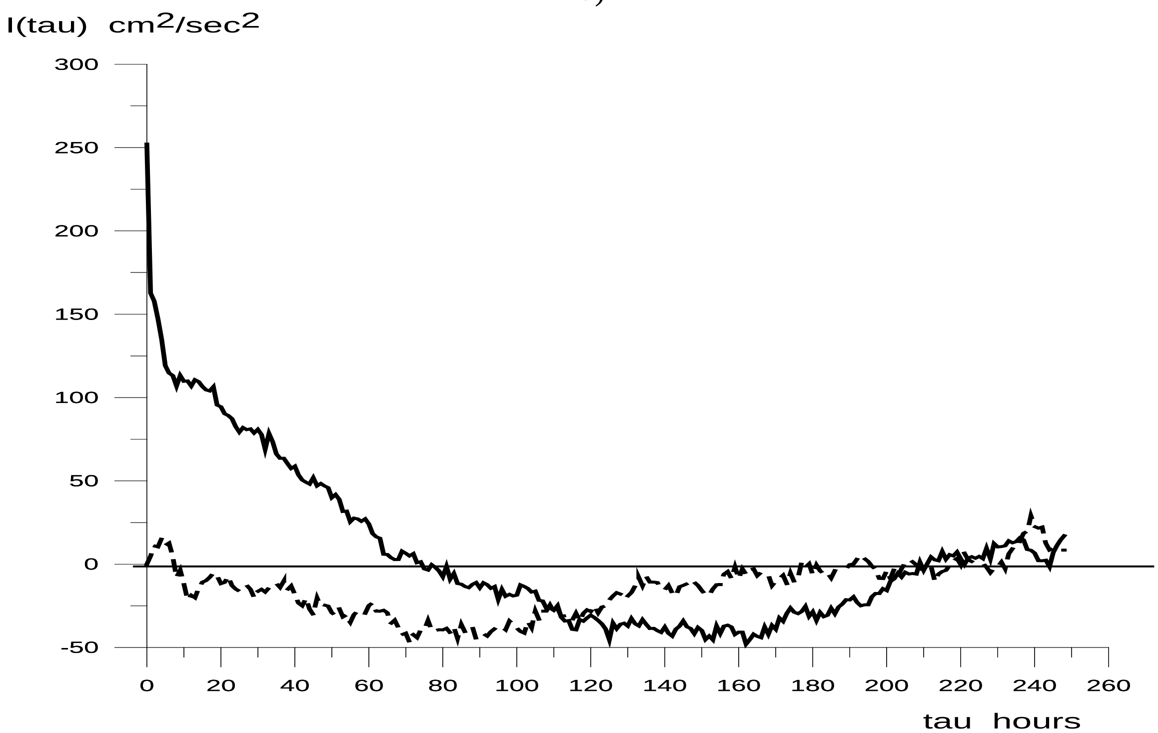
Table 1.
Moment characteristics of current velocity distribution for ABS RS-94

Parameters	Z = 20 m	Z = 25 m
averages:		
v1	1.0 cm/s	2.7 cm/s
v2	9.4 cm/s	8.8 cm/s
$\lvert \mathbf{m^*}_{\mathbf{V}(t)} \rvert$	9.4 cm/s	9.2 cm/s
$\varphi° = \arg \mathbf{m^*}_{\mathbf{V}(t)}$	84°	73°
Variance:		
D_{v1}	165.6 cm^2/s^2	149.2 cm^2/s^2
D_{v2}	86.5 cm^2/s^2	67.6 cm^2/s^2
D_{v1v2}	-13.8 cm^2/s^2	-13.5 cm^2/s^2
$I_1 = D_{v1} + D_{v2}$	252.1 cm^2/s^2	216.8 cm^2/s^2
invariants of tensor of mean square deviation (MSD):		
linear invariant I_1	15.9 cm/s	14.7 cm/s
major semi-axis of MSD ellipse length		
λ_1	13.0 cm/s	12.3 cm/s
minor semi-axis length		
λ_2	9.2 cm/s	8.1 cm/s
orientation of major axis of MSD ellipse		
α	350°	351°
stretchness of MSD ellipse		
$\chi = \lambda_2 / \lambda_1$	0.71	0.66
stability coefficient		
$r_s = I_1^{MSD} / \lvert \mathbf{m}_{\mathbf{V}(t)} \rvert$	1.7	1.6

Table 2.
Spectral characteristics for ABS RS-94, horizon 20 m
(the values on the peaks of spectral invariants); $\Delta\omega$= 0.01267 rad/hour

	Parameters			
Periods (cyclic recurrences), hours (I_1 / Ω)	$I_1(\omega)$, (hour*cm^2/s^2)	$\Omega(\omega)$, (hour*cm^2/s^2)	$\chi(\omega)$, (hour*cm^2/s^2)	$\alpha°(\omega)$
248/496	1516	-668	0.251	349
50/50	107	-15	0.681	282
35/35	69	-22	0.166	353
26/26	79	56	0.376	66
17.7/17.7	70	47	0.428	38
14.6/14.6	74	61	0.319	49
11.8/11.8	48	33	0.497	21
10.1/9.9	45	-26	0.652	332

a)



b)

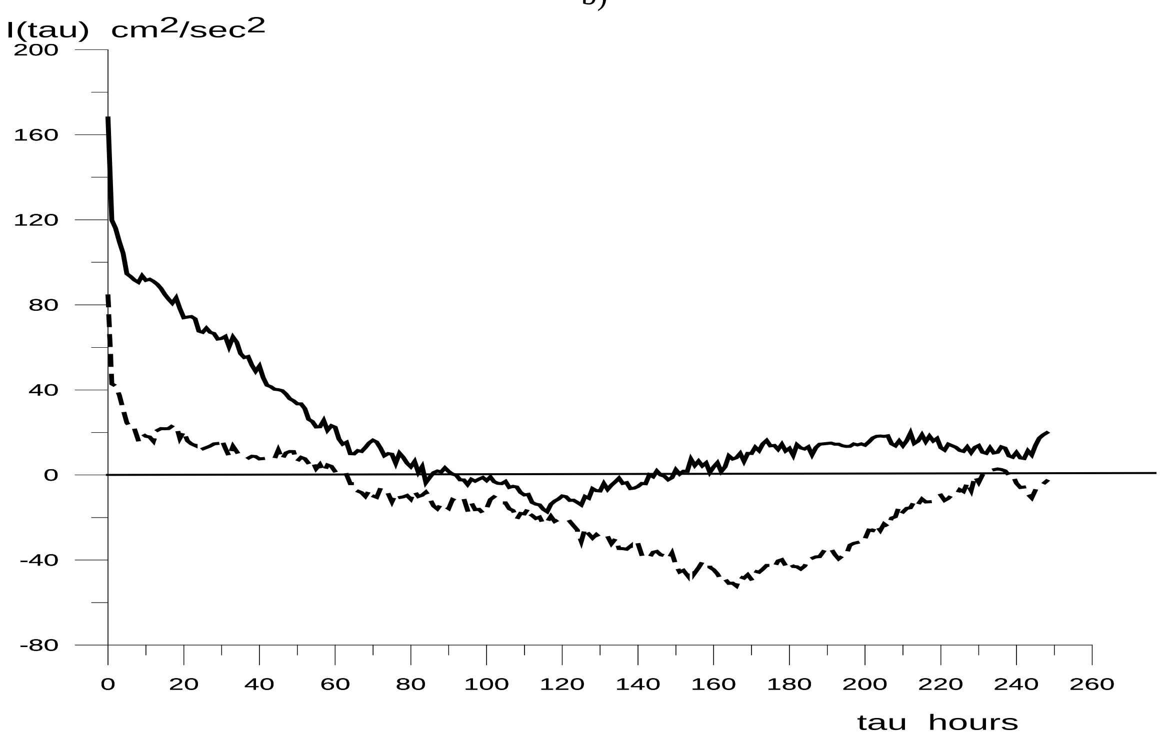
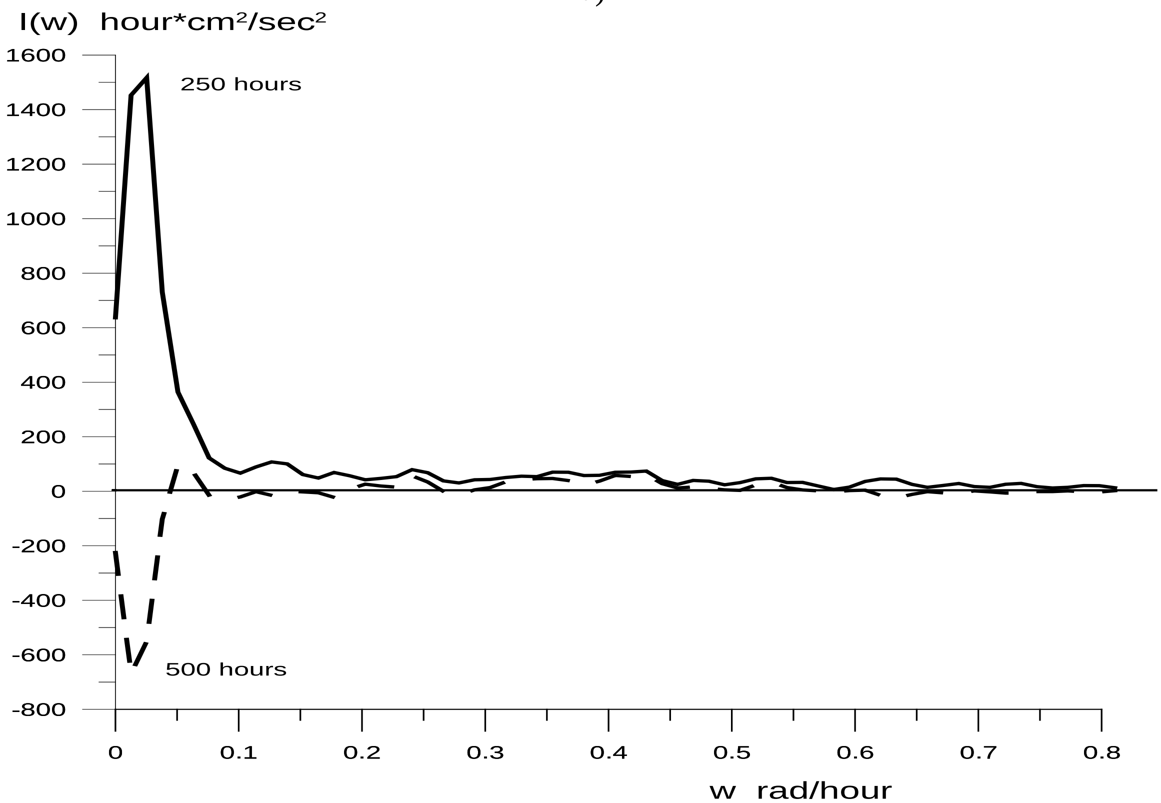


Fig. 5. Invariants of current velocity correlation tensor for ABS RS-94 (RS “Rudolf Samoylovich”, central Baltic, 1987), horizon Z = 20 m.

a) ________ - $I_1(\tau)$, - - - - - - $\Omega(\tau)$; T = 787; Δt = 1 hour; τ_{max} = 248

b) ________ - $\lambda_1(\tau)$, - - - - - $\lambda_2(\tau)$

a)



b)

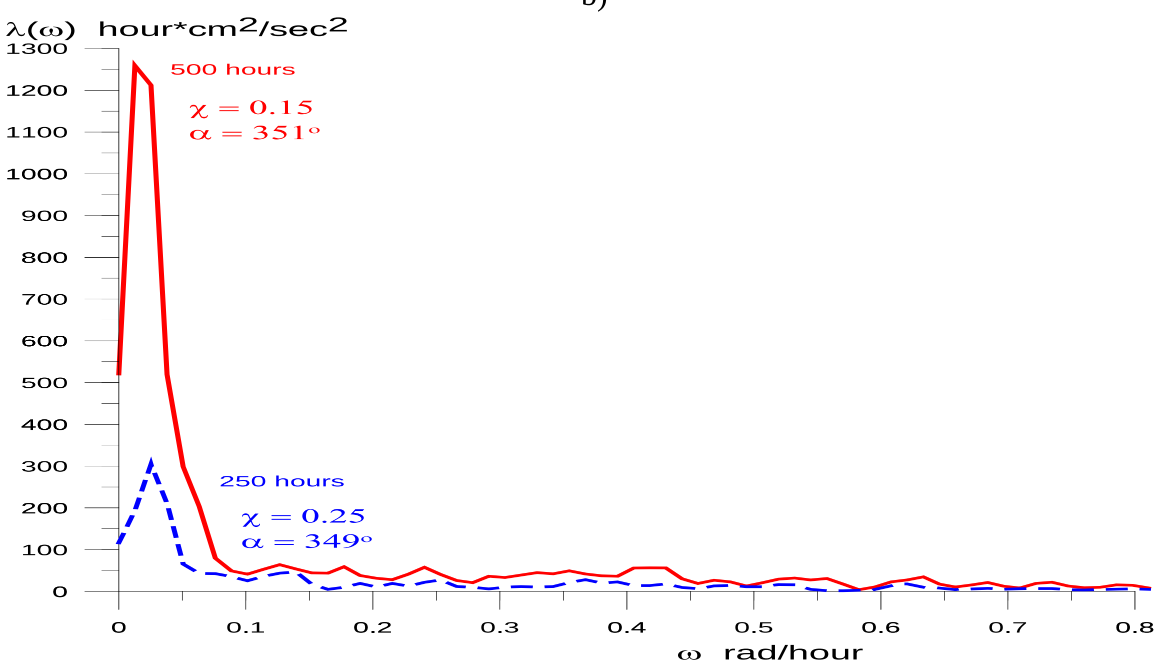


Fig. 6. Invariants of current velocity spectral tensor for ABS RS-94 (RS “Rudolf Samoylovich”, central Baltic, 1987), horizon Z = 20 m.

a) ________ - $I_1(\omega)$, - - - - - - $\Omega(\omega)$; T = 787; Δt = 1 hour; τ_{max} = 248

b) ________ - $\lambda_1(\omega)$, - - - - - $\lambda_2(\omega)$ $\Delta\omega$ = 0.01267 rad/hour

In table 2 are adduced the values of spectral invariants for all peaks of current velocity spectrum. We are seeing that the main contribution in variance of velocity introduces the synoptic fluctuation with cyclic recurrence T_k near 250 hours for $I_1(\omega_k)$ (k=3, ω_k=0,0253 rad/hour) and near 500 hours for $\Omega(\omega_k)$ (k=2, ω_k = 0.0127 rad/hour). This the fact confirms that $\alpha(\omega_2) \approx \alpha^{MSD} \approx 350°$. Although we can see that the fluctuations with this cyclic recurrence (k=2) directed (from one vector to another) counterclockwise and has the intensity near half of general ($\Omega(\omega_{k=2})$ = -668 hour*cm^2/s^2 against $I_1(\omega_{k=3})$ = 1516 hour*cm^2/s^2). It is interesting that the stretchness $\chi(\omega_k)$ of spectral ellipse by ω_k = 0.0253 equal 0.251 (χ^{MSD} = 0.708), and only two spectral ellipses (by T_k = 50 hours and T_k = 10 hours) have a stretchness $\chi(\omega_k)$ near 0.7 . We can say that general stretchness χ^{MSD} is determined by the diversity of these on all peak frequencies and by the diversity of major axes orientation $\alpha(\omega_k)$ on all peak frequencies ω_k.

Note that the peaks of tide fluctuations are removed from the true tide periods (26 hours against 24.8 and 11.8 hours against 12.4); though their rotation from one vector to another is clockwise as for true tide motion, but the ellipses of tide fluctuations have too large stretchness ($\chi(\omega_k)$.= 0.4 – 0.5 against 0.9-0.95 as is wait).

Note although that inertial current fluctuation (T_k = 14.6 hours) on this horizon is expressed rather weakly ($I_1(\omega_k)$ = 74 hour*cm^2/s^2); it has a good rotation ($\Omega(\omega_k)$ = 61 hour*cm^2/s^2) and oriented counterclockwise, but its spectral ellipse is very stretched ($\chi(\omega_k)$ = 0.32 and orientation of ellipse major axis is close to the direction of general transfer ($\alpha(\omega_k)$=49° and φ°=84°).

In table 3 are adduced the statistical estimates of probabilistic characteristics of wind velocity in four regions of the North Sea (these results are taken from the work [23], where the North Sea was divided on four regions). It is seeing from this table:

- the maxima of wind velocity modulus and the general variability (linear invariant I_1) values decrease from north to south, but the most average transfer ($|\mathbf{m^*}_{\mathbf{v}(t)}|$ = 2.6 m/s) is observed in the region II;

- the direction of the average transfer φ° is nearly the same (238-239°), only in reg. I φ°=222°;

- the ellipse of mean square deviations (MSD) in reg. I and reg. II is close to the circle and in reg. II and reg. IV is more stretch;

- the orientation of ellipse major axis in reg. IV and in reg. I is near the direction of average transfer; in reg. II it is perpendicular to φ° and in reg. II it deviates on 40° ;

- the stability coefficient in all regions is more than 1, i.e. the variable part of process prevail.

Table 3.
Moment characteristics of wind velocity distribution
for two regions of the North Sea in 1950-1967 years

Parameters	region I (Shetland Area)	region II (northern North Sea	region III (Central North Sea)	region IV (South North Sea)
Averages (m/s):				
v1	-1.54 m/s	-1.41 m/s	-1.11 m/s	-1.16 m/s
v2	-1.40 m/s	-2.23 m/s	-1.87 m/s	-1.96 m/s
$\|\mathbf{V}\|$	3.41 m/s	3.70 m/s	3.46 m/s	3.21 m/s
$\|\mathbf{V}\|_{min}$	0.1 m/s	0.1 m/s	0.5 m/s	0.4 m/s
$\|\mathbf{V}\|_{max}$	9.9 m/s	9.9 m/s	9.6 m/s	8.4 m/s
$\|\mathbf{m}^*_{\mathbf{V}(t)}\|$	2.1 m/s	2.6 m/s	2.2 m/s	2.3 m/s
$\varphi° = \arg \mathbf{m}^*_{\mathbf{V}(t)}$	222°	238°	239°	239°
Variance:				
D_{v1}	6.1 m^2/s^2	5.4 m^2/s^2	3.3 m^2/s^2	3.5 m^2/s^2
D_{v2}	5.0 m^2/s^2	5.1 m^2/s^2	7.0 m^2/s^2	4.7 m^2/s^2
D_{v1v2}	0.8 m^2/s^2	-0.2 m^2/s^2	-0.6 m^2/s^2	1.1 m^2/s^2
$I_1 = D_{v1}+D_{v2}$	11.05 m^2/s^2	10.44 m^2/s^2	10.26 m^2/s^2	8.12 m^2/s^2
Invariants of tensor of mean square deviation (MSD):				
linear invariant I_1	3.3 m/s	3.2 m/s	3.2 m/s	2.8 m/s
Major semi-axis of MSD ellipse length				
λ_1	2.55 m/s	2.34 m/s	2.66 m/s	2.31 m/s
Minor semi-axis length				
λ_2	2.14 m/s	2.23 m/s	1.79 m/s	1.66 m/s
Orientation of major axis of MSD ellipse				
α	207°	154/334°	279°	240°
Stretchness of MSD ellipse				
$\chi = \lambda_2 / \lambda_1$	0.84	0.96	0.67	0.72
Stability coefficient				
$r_s = I_1^{MSD} / \|\mathbf{m}_{\mathbf{V}(t)}\|$	1.6	1.2	1.5	1.2

So we can say that the properties of the variability of wind velocity are different in different parts of the North Sea.

The graphs of wind velocity spectral invariants are offered for example on fig. 7. It is seeing the presence of one period (annual fluctuation) and of many different cyclic recurrences (2-5 years, 0.5 year and more high-frequency fluctuations) with small amplitudes. The row is too short for to divide two synoptic fluctuations and the annual and 0.8-0.9 year fluctuations (thou the graph of $\lambda_2(\omega)$ has a peak on last frequency); but we are seeing two synoptic fluctuations on the graph of $\Omega(\omega)$ with different direction of rotation from one vector to another – more long (near 5 years) with weak counterclockwise rotation and more short (near 2.5 year) – with clockwise rotation. It is possible to say that on the frequency of half-year fluctuation the regular rotation is absent, and in the high-frequency part of spectrum the rotation counterclockwise is prevailed, so as in the annual fluctuation.

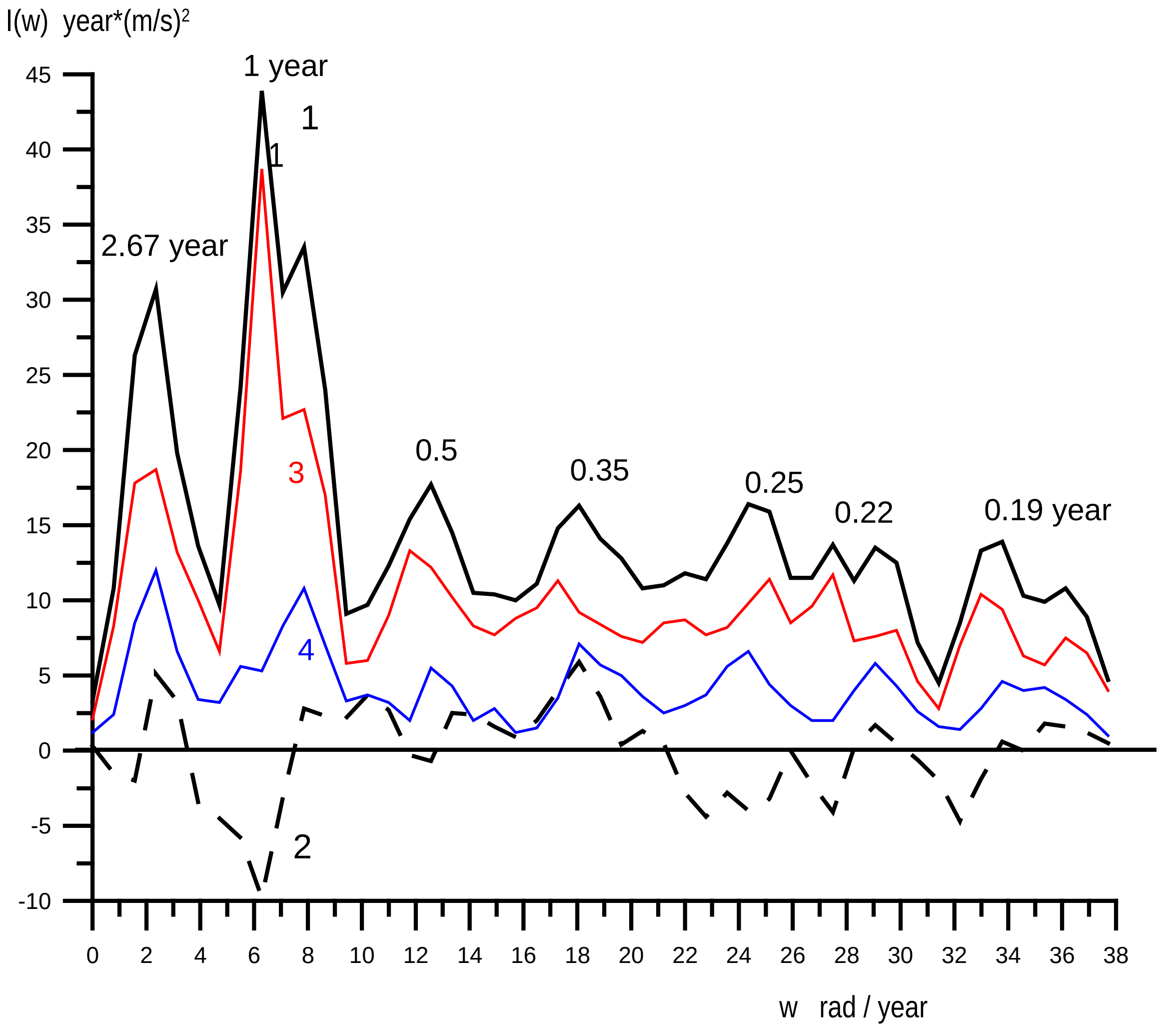


Fig. 7. Invariants of wind velocity spectral tensor for Shetland area of the North Sea, 1950-1967.
1 ____ - $I_1(\omega)$, 2- - - - - - $\Omega(\omega)$;
3 ____ - $\lambda_1(\omega)$, 4 ____ - $\lambda_2(\omega)$.
$T = 216$; $\Delta t = 1$ month; $\tau_{max} = 38$; $\Delta\omega = 0.7854$ rad/year
The numbers near the peaks of spectrum – the values of periods (cyclic recurrences) of wind velocity fluctuations.

Some books had appeared on Russian after the publishing in 1983 the monograph [5] (in this monograph the “vector-algebraic method” has been detailed). In these books above-mentioned method was used by the description of the variability regularities of wind and current velocity.

So in 1989 was published the monograph “The hydromeorological regime investigations of Tallin Bay”. In this book, in particular, the peculiarities of current regime were analyzing [6] by “vector-algebraic method”.

In 1989 was published the book (collected articles) “The regime constituent factors, information base and its analyzing methods” as the methodical manual by the elaboration of 10 volumes reference book “The Seas of USSR. The hydrometeorology and hydrochemistry of the seas of USSR”.

Later, in 2007 was published the monograph [23], where the current regime of Neva Bay was although described on the basis of “vector-algebraic method”.

In early 2000s in Saint-Petersburg Branch of the State Oceanographic Institute (SPb SOI) the algorithm of spectral estimation was elaborated on the basis of the works of M.S. Longuet-Higgins and A.A. Sveshnikov about directional spectrum of sea waves (and some more late works of V.A, Rozhkov and Y.A. Trapeznikov on this subject). Algorithm gave the possibility of calculating of frequency-directional spectrum of scalar and vector hydrometeorological fields. Briefly this algorithm and its application results were published in 2007 in [18]. More detail these results (spectra of sea level and spectral invariants of sea currents in the Gulf of Finland east part) were published in the collected articles “Transactions of SOI, 2007, issue 210. The investigations of the oceans and the seas.” in the article: Yu. Klevantsov, V. Rozhkov. The generalization of Longuet-Higgins – Svshnikov method by the analysis of hydrometeorological fields synoptic variability.

On fig. 8 is offered for example the graph of linear invariant of current velocity frequency-directed spectrum. It was calculated by the data of hydrodynamical simulating of current velocity (program of O. Andreev-A. Sokolov). For the calculating the values of current velocity in the apexes of rectangular triangle are taken and the parameters of directional waves are obtained through the distances between these apexes. It is seeing from the fig. 8 that the current velocity waves are spread on the whole along the parallel (more to the west) and partially along the direction NE-SW (more to SW). The method allows the wave lengths to receive and the speed of their propagation on the frequencies and on the each of 8 compass points for the spectrum peaks.

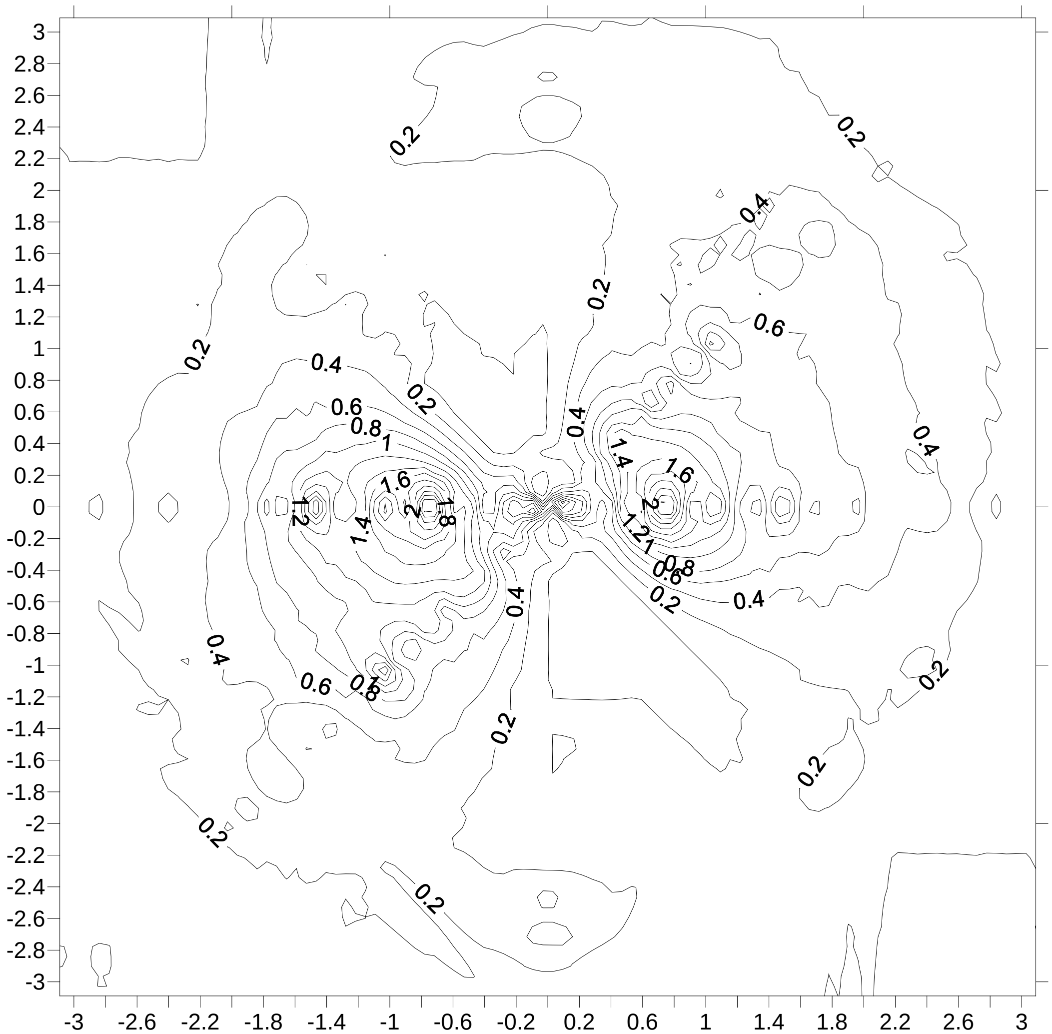


Fig.8. Linear invariant $I_1(\omega,\theta)$ of frequency-directional spectrum of current velocity field in central part of the Baltic Sea (by the results of hydrodynamical simulating in three of the grid points).

Authors choose for this article not accidentally the name consonant to well-known work of A. Defant [29]. We although warn the investigators against illusory ideas about the properties of sea current and wind variability which follow from component-wise approach, the notion of the pair of current velocity projections as the complex number and “rotary component method”. In this article we show that “vector-algebraic method” gives by the analysis of current and wind measurements physically grounded conclusions so far as severely takes into account vector character of velocity and represents the conclusion results in invariant form.

References

1. Абрамов В.М., Клещева Г.П. Использование класса комплексных случайных функций при статистической обработке данных наблюдений над течениями. Межведомственный сборник ЛГМИ, Ленинград, 1978, № 66.
2. Белышев А.П., Рожков В.А. Корреляционный тензор и тензор спектральной плотности как вероятностные характеристики векторных случайных процессов. Труды ГОИН, 1975, вып. 126.
3. Белышев А.П. Клеванцов Ю.П., Микулинская С.М., Рожков В.А. Корреляционный и спектральный анализ скоростей течений в инвариантной форме. Труды ГОИН, 1978, вып. 147.
4. Белышев А.П., Клеванцов Ю.П., Рожков В.А. Об иллюзиях и действительности в методах анализа морских течений. Труды ГОИН, 1982, вып. 157.
5. Белышев А.П. Клеванцов Ю.П., Рожков В.А. Вероятностный анализ морских течений. Ленинград, Гидрометеоиздат, 1983. 264 с.
6. Белышев А.П. Клеванцов Ю.П., Михайлов А.Е. Особенности режима течений Таллинского залива В сб. «Исследования гидрометеорологического режима Таллинского залива». Ленинград, Гидрометеоиздат, 1984, с. 84-100.
7. Белышев А.П., Преображенский Л.Ю. Применение векторно-алгебраического метода для анализа структуры течений Невской губы. В сб. «Режимообразующие факторы, информационная база и методы ее анализа». Ленинград, Гидрометеоиздат, 1989.
8. Боков В.Н., Клеванцов Ю.П., Рожков В.А. Анализ векторных процессов. В сб. «Режимообразующие факторы, информационная база и методы ее анализа». Ленинград, Гидрометеоиздат, 1989.
9. Василенко В.М., Голенко Н.Н. Метод вращательных компонент при исследовании поля течений в океане. В кн. «Комплексные исследования Мирового океана. Тезисы докладов Всесоюзной конференции молодых ученых-океанологов. Актуальные проблемы океанологии.». Т. 2, Москва, 1979.
10. Герман В.Х., Цвецинский А.С. Спектральный анализ векторных временных рядов скоростей морских течений. Метеорология и гидрология, 1977, № 12.
11. Герман В.Х., Цвецинский А.С. Спектральный и взаимный спектральный анализ векторных временных рядов скоростей морских течений. Труды ГОИН, 1979, вып. 144.
12. Горелик Г.С. Колебания и волны. Москва, Физматгиз, 1959, 572 с.
13. Дьедоне Ж. Линейная алгебра и элементарная геометрия. Москва, Наука, 1972, 335 с.
14. Ефимов Н.В., Розендорн Э.Р. Линейная алгебра и многомерная геометрия. Москва, Наука, 1970, 527 с.
15. Казанова Г. Векторная алгебра. Москва, Мир, 1970, 119 с.
16. Каменкович В.М. Основы динамики океана. Ленинград, Гидрометеоиздат, 1973, 240 с.
17. Клеванцов Ю.П., Микалаюнас М.М., Рожков В.А. Сезонная и межгодовая изменчивость скорости ветра. В сб. «Режимообразующие факторы, информационная база и методы ее анализа». Ленинград, Гидрометеоиздат, 1989.
18. Клеванцов Ю.П., Рожков В.А. Метод Лонге-Хиггинса-Свешникова и его обобщение для анализа синоптической изменчивости гидрометеорологических полей. В сб. «Динамика вод Балтийского моря в синоптическом диапазоне пространственно-временных масштабов». Санкт-Петербург, Гидрометеоиздат, 2007, с. 83-95.

19. Кочин Н.Е. Векторное исчисление и начала тензорного исчисления. Москва, Изд-во АН СССР, 1961, 426 с.
20. Ландсберг Г.С. Оптика. Москва, Гостехиздат, 1952, 723 с.
21. Микалаюнас М. Пространственно-временная изменчивость ветра по трассе перехода от портов южной Балтики до Атлантического океана. Диссертация на соискание ученой степени кандидата географических наук. Гидрометцентр СССР, 1989.
22. Озмидов Р.В. Горизонтальная турбулентность и турбулентный обмен в океане. Москва, Наука, 1968, 199 с.
23. Преображенский Л.Ю. Особенности гидрологического режима вершины Финского залива. Санкт-Петербург, Изд-во Политехнического университета, 2007. 122 с.
24. Пугачев В.С. Теория случайных функций. Москва, Физматгиз, 1962, 883 с.
25. Рытов С.М. Введение в статистическую радиофизику. Ч. 1, Москва, Наука, 1976, 496 с.
26. Слуцкий Е.Е. Избранные труды. Теория вероятности. Математическая статистика. Москва, Изд-во АН СССР, 1960, 294 с.
27. Филонов А.Е. Инвариантные спектральные характеристики флуктуаций течения на буйковом полигоне ТРОПЭКС-74. Труды ГОИН, 1979, вып. 146.
28. Co-operative Synoptic Investigation of the Baltic 1964. Conf. Int. Explor. Mer. Service Hydrogr. Copenhagen, 1968, V. 3. Poland, 266 p.
29. Defant A. Reality and illusion in oceanographic survey. J. Mar. Res., 1950, v. 9, № 2.
30. Gonella J. A rotary-component method for analyzing meteorological and oceanographic vector time series. Deep-Sea Res., 1972, v. 19.
31. Hayashi Y. Space-time spectral analysis of rotary vector series. J. Atmos. Sci., v. 36, № 5.

Translation the references from Russian into English:

1. Abramov V., Kletsheva G. Using of complex random function class by statistical processing of data of current measurements. In “Inter-department collected articles of Leningrad State Hydrometeorological Institute”, Leningrad (L.), 1978, N 66.
2. Belyshev A., Rozhkov V. Correlation tensor and spectral density tensor as probabilistic characteristics of vector random processes. Transactions of State Oceanographic Institute (Tr. of SOI), 1975, issue 126.
3. Belyshev A., Klevantsov Y., Mikulinskaja S., Rozhkov V. Correlation and spectral analysis of current velocity in invariant form. Tr. of SOI, 1978, issue 147.
4. Belyshev A., Klevantsov Y., Rozhkov V. About the illusion and reality in sea current analysis methods. Tr. of SOI, 1982, iss. 157.
5. Belyshev A., Klevantsov Y., Rozhkov V. Probabilistic analysis of sea currents. L., Hydrometeoizdat, 1983, 264 p.
6. Belyshev A., Klevantsov Y., Mikhailov A. The peculiarities of current regime in Tallin Bay. In the book” “The hydrolmeteorological regime investigations of Tallin Bay’, L., Hydrometeoizdat, 1984, pp. 84-100.
7. Belyshev A., Preobrazhensky L. The application of vector-algebraic method by current structure analysis of Neva Bay. In the collected articles “The regime constituent factors, information base and its analysis methods”, L., Hydrometeoizdat, 1989.
8. Bokov V., Rozhkov V, The vector process analysis. “The regime constituent factors, information base and its analysis methods”, L., Hydrometeoizdat, 1989.

9. Vasilenko V., Golenko N. The rotary component method by the current field investigation in the ocean. In the collected articles “Complex investigations of the World Ocean”, V.2. Moskow (M.), 1979.
10. German V., Tsvetsinsky A. Spectral analysis of vector time series of sea current velocities. Hydrology and meteorology, 1977, N 12.
11. German V., Tsvetsinsky A. Spectral and cross-spectral analysis of vector time series of sea current velocities. Tr. of SOI, 1979, iss. 144.
12. Gorelik G. Fluctuations and waves. M., Physmatgiz, 1959,572 p.
13. Diedonnet J. Linear algebra and elementary geometry. M., Science, 1972, 335 p.
14. Efimov N., Rosendorn E. Linear algebra and multi-dimensional geometry. M., Science, 1970, 527 p.
15. Kazanova G. Vector algebra. M., World, 1970,119 p.
16. Kamenkovich V. The foundations of ocean dynamics. L., Hydrometeoizdat, 1973, 240 p.
17. Klevantsov Y., Mikalajunas M., Rozhkov V. The seasonal and inter-annual variability of wind velocity. In the collected articles “The regime constituent factors, information base and its analysis methods”, L., Hydrometeoizdat, 1989.
18. Klevantsov Y., Rozhkov V. Longuet-Higgins - Sveshnikov method and its extension by the synoptic variability analysis of hydrometeorological fields. In the book “The Baltic Sea water dynamics in synoptic range of space-time scales”, Saint-Petersburg, Hydrometeoizdat, 2007, pp. 83-95.
19. Kochin N. Vector calculus and the principles of tensor calculus. M.. Acad. of Science Publishing House, 1961, 426 p.
20. Landsberg G. The optics. M., Gostekhizdat, 1952, 723 p.
21. Mikalajunas M. The space-time variability of wind along the route of going from the ports of the south Baltic to the Atlantic Ocean. Dissertation for a Candidate’s degree of geographical sciences. Hydrometcenter of USSR, M., 1989.
22. Ozmidov R. Horizontal turbulence and turbulent water change in the ocean. M., Science, 1968, 199 p.
23. Preobrazhensky L. The peculiarities of hydrological regime of the Gulf of Finland summit. Saint-Petersburg, Polytechn. University Publishing House, 2007, 122 p.
24. Pugachev V. The theory of random functions. M., Physmatgiz, 1962, 883 p.
25. Rytov S. The introduction in statistical radio-physics. Part I. M., Science, 1976, 496 p.
26. Slutsky E. Selected works. Probability theory. Mathematical statistics. M., Acad. of Sci. Publishing House, 1960, 294 p.
27. Filonov A. Invariant spectral characteristics of current fluctuations on the buoy polygon TROPEX-74. Tr. of SOI, 1979, iss. 146.